New results on symmetric quantum cryptanalysis

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New Results on Symmetric Quantum Cryptanalysis

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Outline

- Introduction
  On Quantum-Safe Symmetric Cryptography

- Efficient Quantum Collision Search
  joint work with A. Chailloux and A. Schrottenloher
  [Asiacrypt17]

- Efficient Quantum k-XOR search
  joint work with L. Grassi and A. Schrottenloher

Symmetric Cryptography

Classical Cryptography

Enable secure communications even in the presence of malicious adversaries.

Asymmetric (e.g. RSA) *(no key exchange/computationally costly)*

Security based on well-known hard mathematical problems (e.g. factorization).

Symmetric (e.g. AES) *(key exchange needed/efficient)*

Ideal security defined by generic attacks ($2^{|K|}$).

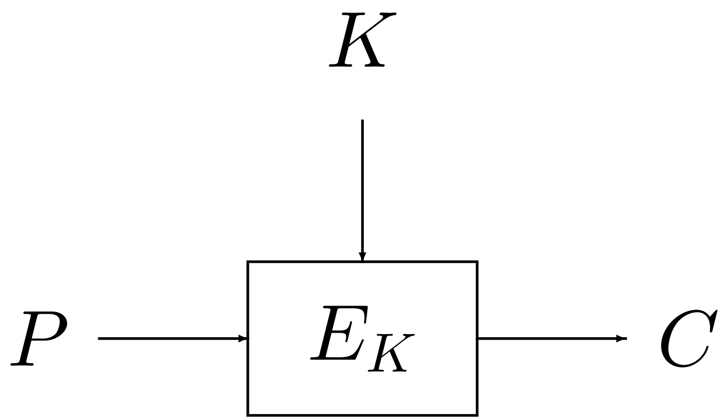
Need of continuous security evaluation (cryptanalysis).

$\Rightarrow$ Hybrid systems! (e.g. in SSH)

Symmetric primitives

- Block ciphers, (stream ciphers, hash functions..)

Message decomposed into blocks, each transformed by the same function E_K.



E_K is composed of a round transform repeated through several similar rounds.

Generic Attacks on Ciphers

- Security provided by an ideal block cipher defined by the best generic attack:
  exhaustive search for the key in $2^{|K|}$.

- Recovering the key from a secure cipher must be infeasible.

  $\Rightarrow$ typical key sizes $|K| = 128$ to 256 bits.

Cryptanalysis: Foundation of Confidence

Any attack better than the generic one
is considered a "break".

- Proofs on symmetric primitives need to make unrealistic assumptions.
- We are often left with an empirical measure of the security: cryptanalysis.
- Security redefinition when a new generic attack is found (e.g. accelerated key search with bicliques [BKR 12])

Current scenario

- Competitions (AES, SHA-3, eSTREAM, CAESAR).
- New needs: lightweight, FHE-friendly, easy-masking.
  $\Rightarrow$ Many good proposals/candidates.
- How to choose?
- How to be ahead of possible weaknesses?
- How to keep on trusting the chosen ones?

Cryptanalysis: Foundation of Confidence

When can we consider a primitive as secure?

- A primitive is secure as far as no attack on it is known.
- The more we analyze a primitive without finding any weaknesses, the more reliable it is.

Design new attacks + improvement of existing ones:

- essential to keep on trusting the primitives,
- or to stop using the insecure ones!

On weakened versions

If no attack is found on a given cipher, what can we say about its robustness, security margin?

The security of a cipher is not a 1-bit information:

- Round-reduced attacks.
- Analysis of components.

$\Rightarrow$ determine and adapt the security margin.

On high complexities

When considering large keys, sometimes attacks breaking the ciphers might have a very high complexity far from practical *e.g.*. 2^{120} for a key of 128 bits.

Still dangerous because:

- Weak properties not expected by the designers.
- Experience shows us that attacks only get better.
- Other existing ciphers without the "ugly" properties.

▶ When determining the security margin: find the highest number of rounds reached.

Post-Quantum Symmetric Cryptography

Post-Quantum Cryptography

Adversaries have access to **quantum computers.**

Asymmetric (e.g. RSA):

Shor's algorithm: Factorization in polynomial time

$\Rightarrow$ current systems not secure!

Solutions: lattice-based, code-based cryptography...

Symmetric (e.g. AES):

Grover's algorithm: Exhaustive search from $2^{|K|}$ to $2^{|K|/2}$.

Double the key length for equivalent ideal security.

We don't know much about cryptanalysis of current ciphers when having quantum computing available.

Post-Quantum Cryptography

Problem for present existing long-term secrets.
$\Rightarrow$ start using quantum-safe primitives NOW.

Important tasks:

- Conceive the cryptanalysis algorithms for evaluating the security of symmetric primitives in the P-Q world.
- Use them to evaluate and design symmetric primitives for the P-Q world.

Quantum Symmetric Cryptanalysis

Some recent results on Q-symmetric crytanalysis:

3-R Feistel [Kuwakado-Morii10], Even-Mansour [Kuwakado-Morii12], Mitm [Kaplan14], Related-Key [Roetteler-Steinwandt15], Diff-lin [Kaplan-Leurent-Leverrier-NP16], Simon's[Kaplan-Leurent-Leverrier-NP16], FX [Leander-May17], parallel multi-preim. [Banegas-Bernstein17], Multicollision [Hosoyamada-Sasaki-Xagawa17],
AEZ [Bonnetain17], Modular additons [Bonnetain-NP18]...

Quantum Symmetric Cryptanalysis

Two main models used:

- Q1:
  classical queries and access to a quantum computer.

- Q2:
  +superposition queries to a quantum cryptog. oracle.

Very powerful, BUT...

Q2: Superposition Model

Many good reasons to study security in this scenario:

- Simple
- Non-trivial: Many constructions still seem resistant: AES, SALSA20, NMAC, HMAC...
- Inclusive of all intermediate scenarios

Defined and used in: [Zhandry12], [Boneh-Zhandry13], [Damgård-Funder-Nielsen-Salvail13], [Mossayebi-Schack16], [Song-Yun17], Simon's attacks, FX, AEZ...

An attack in this model $\Rightarrow$ might not be safe to implement the primitive in a quantum computer.

On Quantum attacks

- Compare to best generic attack,
- generic attack is accelerated, so
- broken classical primitive might be unbroken in a quantum setting.

Collision Search

w. A. Chailloux & A. Schrottenloher

Collision Search Problem

Given a random function $H : \{0,1\}^n \rightarrow \{0,1\}^n$, find $x, y \in \{0,1\}^n$ with $x \neq y$ such that $H(x) = H(y)$.

Many applications: *i.e.* generic attacks on hash functions.

(Multi-preimage search can be seen as a particular case).

Best known algorithms

	Time	Queries	Memory
Pollard's rho	$2^{n/2}$	$2^{n/2}$	$poly(n)$
Parallelization (2^s)	$2^{n/2-s}$	$2^{n/2}$	2^s

	Time	Queries	Qubits
Grover	$2^{n/2}$	$2^{n/2}$	$poly(n)$
BHT	$2^{2n/3}*$	$2^{n/3}$	$poly(n)*$
Ambainis	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$

Considered Model

- The same one as in all the previous quantum algorithms BUT we limit the amout of quantum memory available to a small amount $poly(n)$.

- Available small quantum computers seems like the most plausible scenario.

- We are interested in the theoretical algorithm and we did not take into account implementation aspects.

Starting Point: BHT Algorithm

- Optimal number of queries,
- $poly(n)$ qbits,
- But time?

BHT: Summarized procedure

- Build a list L of size $2^{n/3}$ elements (classic memory),

- Exhaustive search for finding one element that collides: With AA, the number of iterations is $(\frac{2^n}{2^{n/3}})^{1/2} = 2^{n/3}$.

  Testing the membership with L for the superposition of states costs $2^{n/3}$ with n qbits:

$$\text{Time: } 2^{n/3} + 2^{n/3}(1 + 2^{n/3}) \approx 2^{2n/3}$$

Can we improve this?

Lets build the list L with distinguished points
e.g. $H(x_i) = 0^u || z$, for $z \in \{0,1\}^{n-u}$.

The cost of building the list is bigger: $2^{n/3+u/2}$.
The setup of AA is bigger: $2^{u/2}$
The membership test stays the same: $|L| = 2^{n/3}$
BUT The number of iterations is smaller: $2^{n/3-u/2}$

Time: $2^{n/3+u/2} + 2^{n/3-u/2}(2^{u/2} + 2^{n/3}) \approx 2^{2n/3-u/2} + 2^{n/3+u/2}$

With optimal parameters

The cost will be optimized for a certain size of L: $2^v \neq 2^{n/3}$.

Time: $2^{v+u/2} + 2^{\frac{n-v-u}{2}}(2^{u/2} + 2^v)$

For $v = n/5$, $u = 2n/5$: Time: $\tilde{O}(2^{2n/5})$

For multiple preimage search, the algorithm is similar, but we only keep in L the distinguished points amongst the already given ones.

Comparison

	Time	Queries	Qubits	Classic Memory
Pollard	$2^{n/2}$	$2^{n/2}$	0	$poly(n)$
Grover	$2^{n/2}$	$2^{n/2}$	$poly(n)$	0
BHT	$2^{2n/3}$	$2^{n/3}$	$poly(n)$	$2^{n/3}$
Ambainis	$2^{n/3}$	$2^{n/3}$	$2^{n/3}$	0
New algorithm	$2^{2n/5}$	$2^{2n/5}$	$poly(n)$	$2^{n/5}$

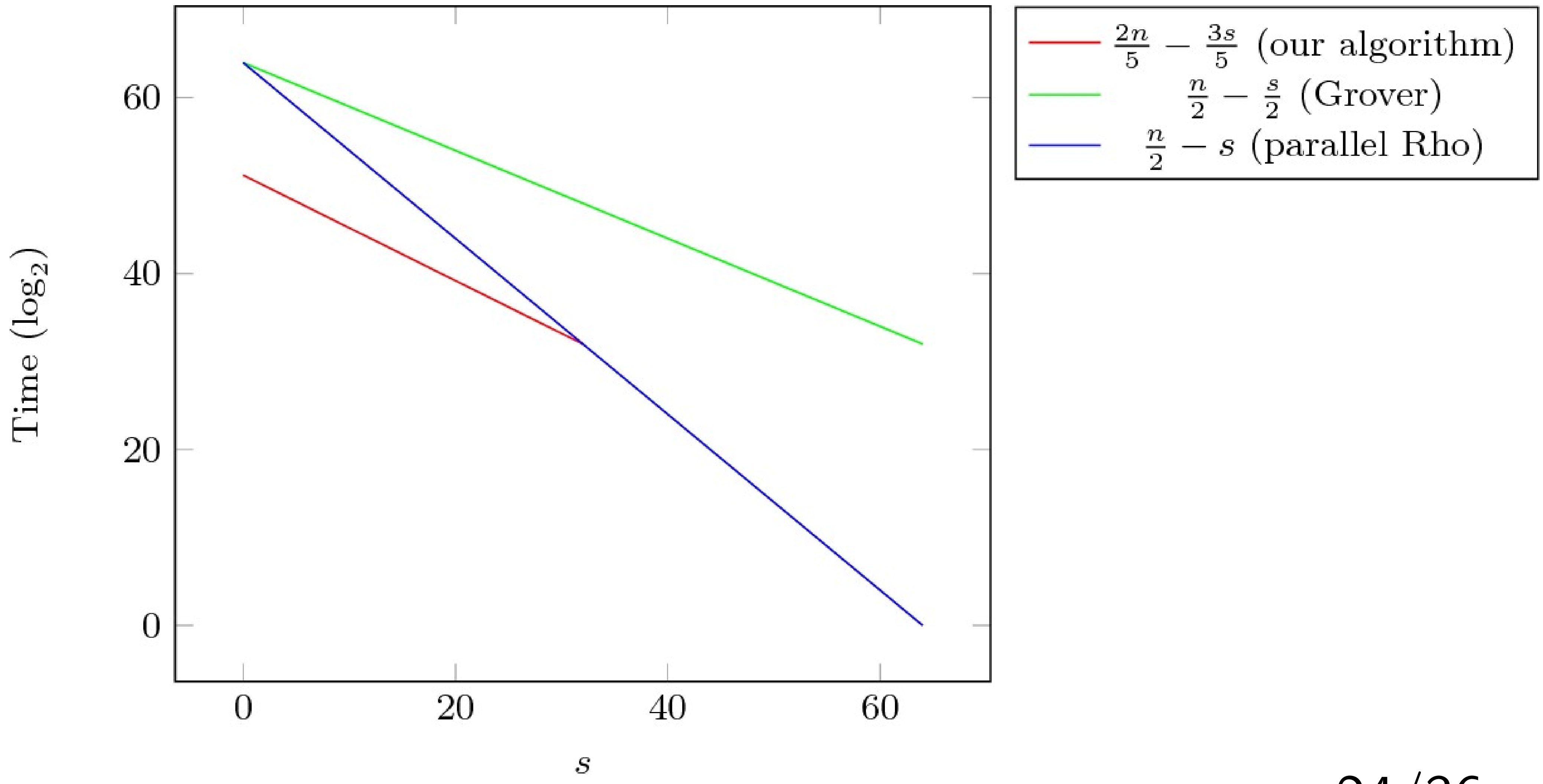
Parallelization

With 2^s n-qbit registers and "external" parallelization we can achieve:

Time: $2^{v+u/2-s} + 2^{\frac{n-v-u}{2}-s/2}(2^{u/2} + 2^v)$

Our theoretical algorithm seems more efficient than classical parallelization/Beal up to $s = n/4$

Comparison example: n=128



Example of Applications (1)

- 1. Hash functions: Collision and Multi-preimages time from $2^{n/2}$ to $2^{2n/5}$ and $2^{3n/7}$ (Q1).
  Ex.- time and queries for $n = 128$:
  rho= 2^{64}, ours= $2^{51.2}$ (with less than 1GB classical)

- 2. Multi-user setting: Recover Ctxt, from same Ptxt, 2^t different keys: apply multi-preimage algorithm (Q1). Depending on the value of t different gain.

Example of Applications (2)

- 3. Operation modes: Collision attacks on CBC: 2^t Ctxt, find one preimage $\Rightarrow$ Ptxt. (Q2). If frequent rekeying (Q1).
- 4. Bricks for Cryptanalysis: Collision, multi-preimage search: often bricks of more technical cryptanalysis: improve the steps.

Conclusion 1

New efficient collision search algorithm with small quantum memory.

Many applications in symmetric cryptograhy.

Open question: is it possible to meet the optimal $2^{n/3}$ in time with small quantum memory? (Quantum random walks, quantum learning graphs...?)

Quantum Efficient Algorithms for the k-XOR Problem

w. L. Grassi & A. Schrottenloher

k-XOR problem with random functions

Given query access to a random function $H : \{0,1\}^n \rightarrow \{0,1\}^n$, find $x_1, \ldots, x_k$ such that $H(x_1) \oplus \ldots \oplus H(x_k) = 0$.

For us, equivalent to the case with k different random functions.

Many applications (with k-SUM, similar algorithms apply), ex.: attacks on FSB, XLS, SWIFFT; correlation attacks.

The 3-XOR problem

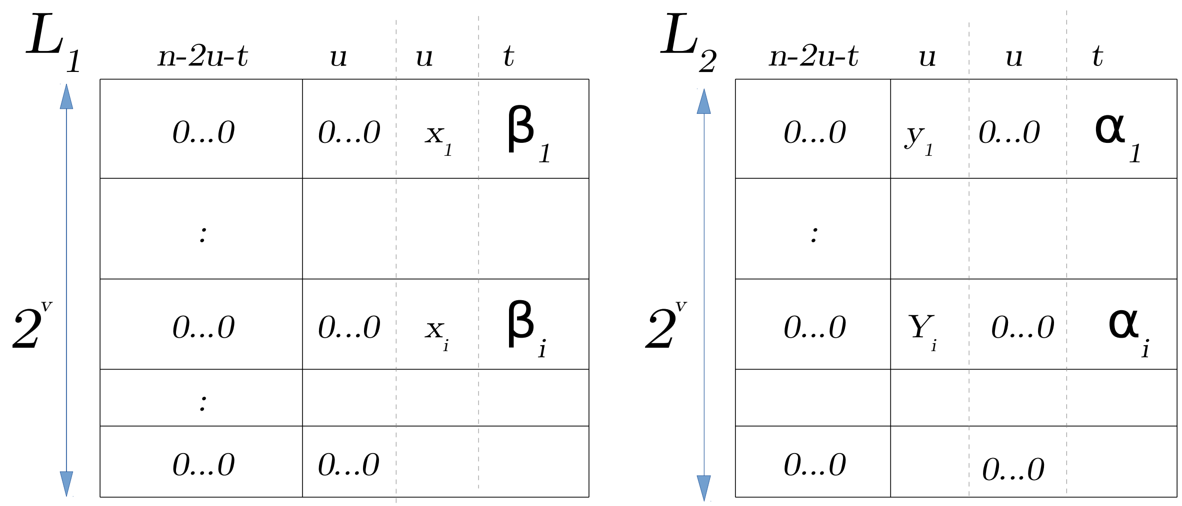
Find 3 elements that XOR to 0: not much better than collision in classical setting.

Classically, no exponential acceleration, only logarithmic factors:

Complexity of about $2^{n/2}$ with out this factors.

3-XOR: Low Quantum Memory Algorithm

- 1st approach, distinguished point: $2^v = 2^{n/8}$, $T = 2^{3n/8}$
- 2nd approach, techniques linked to "list merging":



Improved time$= 2^{5n/14}$, with $2^v = 2^{n/7}$.

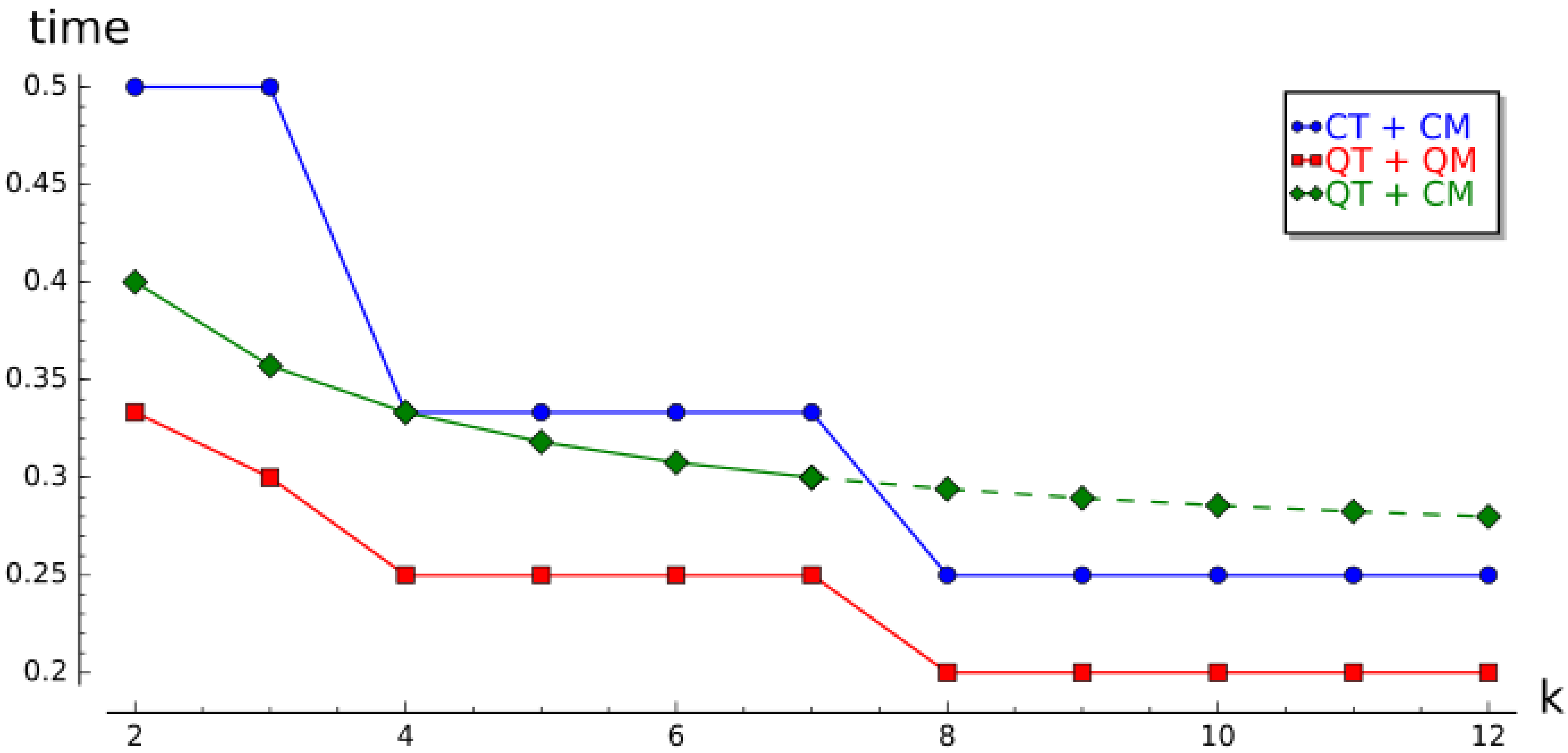
- More efficient than collision, contrary to classical!

3-XOR: High Quantum Memory Algorithm

- Same technique as before, but no need for the positions to '0' in both lists.
- Complexity of:
  $2^{v+u/2} + 2^{\frac{n-2v}{2}}(2^{v-u})$.
- This becomes optimal for
  QM$= 2^{n/5}$ and Time$= 2^{3n/10}$.

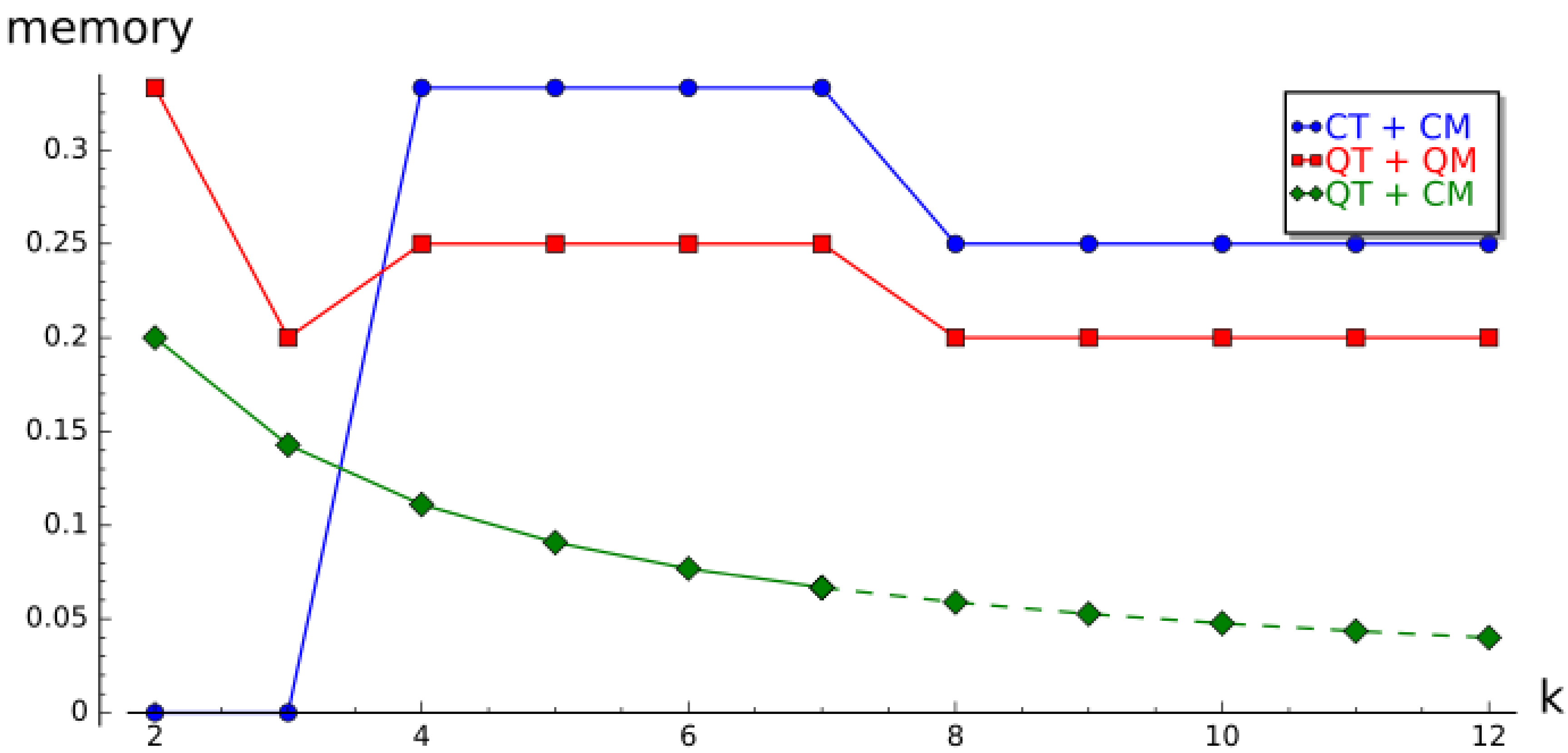
The k-XOR algorithms

Similar algorithms can be applied to other values of k



The k-XOR algorithms

Similar algorithms can be applied to other values of k



Conclusion 2

- We have shown that quantum 3-xor problem is exponentially easier that the quantum collision problem (in both settings), contrary to classical.
- The complexity of solving the 3-xor problem with allowed quantum memory beats the lower bound for quantum collision of $2^{n/3}$
- For generic k, low quantum memory improves Wagner up to $k = 8$, and allowed quantum memory for all k.

Final Conclusion

Open problems

- Optimal collision time $2^{n/3}$?.
- Algebraic attacks
- Boomerang attacks
- FSE Stevens: Quantum cryptanalysis of SHA-2?
- AES quantum evaluation- on going work.
- Generic key-length extensions?
- What about state size? ...

Lots of things to do !